Starting with

$$P_0 = \sum_{i=1}^{m} \beta^i (A_i - s_1 P_i), \tag{1}$$

we can first differentiate the right-hand side keeping R_1 (hence β) *fixed*. This will give us one of the two terms that we'll need when we go back to compute the full derivative using the product rule. Assume that $j > 1$.

$$\begin{aligned}
\frac{d\,\text{RHS}}{dA_{1\to j}} &= \sum_{i=1}^{m} \beta^i \frac{d}{dA_{1\to j}} \left[A_i - s_1 \left((1+r)^i P_0 - \sum_{k=1}^{i} (1+r)^{i-k} A_k \right) \right] \\
&= \sum_{i=1}^{m} \beta^i \frac{dA_i}{dA_{1\to j}} + s_1 \sum_{i=1}^{m} \beta^i \sum_{k=1}^{i} (1+r)^{i-k} \frac{dA_k}{dA_{1\to j}} \\
&= -\beta + \beta^j (1+r)^{j-1} + s_1 \sum_{i=1}^{m} \beta^i (1+r)^{i-1}(-1) + s_1 \sum_{i=j}^{m} \beta^i (1+r)^{i-j} (1+r)^{j-1} \\
&= -\beta + \beta^j (1+r)^{j-1} - s_1 \sum_{i=1}^{m} \beta^i (1+r)^{i-1} + s_1 \sum_{i=j}^{m} \beta^i (1+r)^{i-1} \\
&= -\beta + \beta^j (1+r)^{j-1} - s_1 \sum_{i=1}^{j-1} \beta^i (1+r)^{i-1}
\end{aligned}$$

Now, differentiating both sides of (1) yields

$$0 = \frac{1}{12} \sum_{i=1}^{m} (-i) \beta^{i+1} \frac{dR_1}{dA_{1\to j}} (A_i - s_1 P_i) + \frac{d\,\text{RHS}}{dA_{1\to j}}$$

which may be rearranged to

$$\frac{dR_1}{dA_{1\to j}} = \frac{-\beta + \beta^j (1+r)^{j-1} - s_1 \sum_{i=1}^{j-1} \beta^i (1+r)^{i-1}}{\frac{1}{12} \sum_{i=1}^{m} i \beta^{i+1} (A_i - s_1 P_i)}.$$

Finally, note that the summation in the last term in the numerator may be simplified to

$$\sum_{i=1}^{j-1} \beta^i (1+r)^{i-1} = \beta \sum_{i'=0}^{j-2} [\beta(1+r)]^{i'} = \beta \frac{1 - [\beta(1+r)]^{j-1}}{1 - \beta(1+r)}$$